Stochastic Trends, Deterministic Trends and Business Cycle Turning Points

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This study examines the relationship between specifications for long-run output patterns and specifications for business cycle dynamics. In an application to US GDP, it is found that inferences about the nature of the trend in output are not robust to changes in the specification for short-run fluctuations. Similarly, the choice of which model best describes the transitory movements in output depends on the way in which the trend is specified.

The empirical analysis makes use of Bayesian methods to compare non-nested models for US GDP. Inspection of the predictive likelihoods for the individual data points suggests that the information contained in the data is largely limited to the observations associated with business cycle turning points.

1 Introduction

Any empirical study of long-run output patterns or of business cycle[1] dynamics is conditioned on the way in which the analyst chooses to separate the trend from the stationary component of the observed series. In their study of the nature of the trend in output and in other macroeconomic time series, Nelson and Plosser (1982) make use of a linear time series specification to model the stationary dynamics that were not of primary interest; virtually all[2] of the subsequent work on the nature of the trend in output has adopted this convention.

Although linear models of business cycles are a relatively straightforward way of modelling stationary series, these models appear to be unable to explain some of the basic features of observed business cycles. For example, one prediction of linear models is that business cycles should be symmetric; empirical evidence[3] suggests that recessions are generally shorter than expansions. Recent work in non-linear models of the business cycle[4] has provided several models that are easily estimated and which are capable of reproducing the business cycle features of output data. In concentrating on the business cycle features of output data, most work[5] on non-linear time series models has made use of a stochastic trend to identify the stationary component of output fluctuations.

In any modelling strategy, simplifying assumptions about the phenomena that are not of immediate interest are inevitable, and indeed desireable. Nonetheless, it is equally desirable to ensure that conclusions about the phenomenon of interest are robust to changes in the simplifying assumptions. The purpose of this study is to examine the role of the auxiliary hypotheses that are commonly used in time series models for output:

i Using linear time series models to represent the stationary component of output in studies of the trend in output.

ii Using a stochastic trend to represent the non-stationary component of output in studies of the business cycle features of output.

It is found that *neither assumption is innocuous.* Conclusions about the nature of the trend in US GDP are reversed if non-linear business cycle specifications are taken into consideration, and

[1] The term "business cycle" is used in this paper to refer to the short-run fluctuations of output around trend.

[2] For example, linear models are used in each of the 15 articles in the symposium issue of *Econometric Theory*, Vol 10, Nos. 3/4 on Bayesian methods and unit roots

[3] See, for example Neftçi (1984).

[4] Hamilton (1989), Beaudry and Koop (1993), Potter (1994)

[5] Including all the references to the non-linear time series literature cited in this paper.

conclusions about the most appropriate model for the business cycle are are altered if a deterministic trend is not ruled out.

The empirical application presented below makes use of Bayesian methods of inference to calculate posterior model probabilities conditional on the various auxiliary hypotheses. Since conclusions about the nature of both the trend and the business cycle components of output are not robust to changes in the auxiliary assumptions, marginal model probablities are also presented. In the discussion of the results, the paper adopts Geweke's (1994) recommendation of reporting the predictive likelihoods for each model and for each observation. An interesting feature of this approach is that it is possible to evaluate the marginal contribution of each observation in determining the appropriate model for the trend and/or the business cycle. Inspection of the individual predictive likelihoods suggests that observations at or near business cycle turning points play an extremely important role in model selection.

The paper has 5 sections. Section 2 describes the models under consideration, Section 3 outlines the procedure used for evaluating the models, Section 4 presents the results and Section 5 concludes.

2 Models of Trends and Business Cycles

In this section, the models under consideration are presented; since these models are well-known, they are described only briefly. The first two models use the conventional linear time series specification for business cycle movements. The linear model with a deterministic trend can be written as:

$$(1-\rho(L))(y_t-\beta t)=\alpha+\varepsilon_t \tag{1}$$

where y_t is the log of output in period t, $\rho(L)$ is a polynomial in the lag operator of order r, β is the rate of growth of the trend, α is the intercept for the log of output and ε_t is an *iid* $N(0,\sigma^2)$ error term. In order to make clear the distinction between the non-stationary trend and the stationary cycle, suppose that the roots of the polynomial $1-\rho_1 L-\rho_2 L^2-\cdots-\rho_r L^r=0$ are all outside the unit circle. Since this is the case, fluctuations in ε_t have only temporary effects; long-run forecasts for the log of output will converge to a deterministic trend with growth rate β.

The linear model with a stochastic trend takes the form:

$$(1-\rho(L))(y_t-y_{t-1})=\beta+\varepsilon_t \tag{2}$$

It is well-known that the existence of a unit root in the autoregressive process for y_t implies that fluctuations in ε have permanent effects, even if the the roots of $1 - \rho_1 L - \rho_2 L^2 - \ldots - \rho_r L^r = 0$ are outside the unit circle. Although forecasts for the rate of growth will eventually converge to a constant, there is no tendancy for the level of output to return to any particular value.

In his non-linear framework for analyzing business cycles, Hamilton (1989) introduces a latent random variable $s_t \in \{0, 1\}$, where $s_t = 1$ is associated with expansions and where $s_t = 0$ is associated with recessionary periods. The state variable s_t is assumed to follow a first order Markov process, with the matrix of transition probabilities defined according to:

$$\begin{bmatrix} p(s_t = 0|s_{t-1} = 0) & p(s_t = 1|s_{t-1} = 0) \\ p(s_t = 0|s_{t-1} = 1) & p(s_t = 1|s_{t-1} = 1) \end{bmatrix} = \begin{bmatrix} a & 1-a \\ 1-b & b \end{bmatrix}$$

Hamilton (1989) and others[6] find that models with this structure provide encouraging results when applied to output data.

The Markov-switching structure is incorporated into the deterministic trend model by supposing that the conditional mean of the log of output alternates between two deterministic trends:

$$(1 - \rho(L))(y_t - \beta t) = \alpha_0 + \alpha_1 s_t + \varepsilon_t \tag{3}$$

where α_1 is assumed to be greater than 0 to identify $s_t = 1$ as an expansionary period. In this model, the log of output "jumps" between two parallel trends; recessions are characterised by periods in which the economy is described by the trend associated with consistently lower levels of output.

The Markov-switching model with stochastic trends is similar to that used by Hamilton (1989):

$$(1 - \rho(L))(y_t - y_{t-1}) = \beta_0 + \beta_1 s_t + \varepsilon_t \tag{4}$$

where again β_1 is assumed to be greater than 0 to identify $s_t = 1$ as an expansionary period. In this case, the economy alternates between periods of high growth and of low growth.

Another simple non-linear model that has been sucessfully applied to output data is the Threshold Autoregressive (TAR) model presented by Beaudry and Koop (1993). In their simple model, the assymetric nature of the business cycle is incorporated the the variable CDR_t (Current Depth of

[6] See the survey in the July 1994 issue of the *Journal of Business and Economic Statistics.*

Recession):

$$CDR_t \equiv y_t - \max\{y_t, y_{t-1}, y_{t-2}\ldots\}$$

If the economy is in recession, then CDR_t will be negative. If the economy is expanding, CDR_t will typically be zero, except perhaps near the beginning of the upturn. An important feature of business cycles that is not well-described by linear models is that recessions are typically shorter than expansions. The TAR model captures this feature of the data by using CDR_{t-1} as an explanatory variable for current output. If the coefficient for CDR_{t-1} is negative, then output will expand more rapidly than would have been the case if the economy were in expansion, and the recession would be shortened. The TAR model with a deterministic trend is expressed as:

$$(1-\rho(L))(y_t - \beta t) = \alpha_0 + \alpha_1 CDR_{t-1} + \varepsilon_t \tag{5}$$

Beaudry and Koop (1993) study the TAR model with a stochastic trend:

$$(1-\rho(L))(y_t - y_{t-1}) = \beta_0 + \beta_1 CDR_{t-1} + \varepsilon_t \tag{6}$$

The literature on non-linear time series models offers many extensions to the simple Markov-switching models (3) and (4)[7] as well as the TAR models (5) and (6)[8]. The simple models studied here were chosen mainly for the relative ease with which they can be estimated as well as the simple way in which they can be compared to both each other and the linear model.

3 Model Selection

3.1 Elicting Proper Priors

Nelson and Plosser's (1982) finding in favour of the stochastic trends model generated a great deal of activity in the econometrics literature. Much of the literature centred on the theoretical problems posed in identifying the properties of classical estimators when the underlying data generating processes are non-stationary - see chapter 20 of Davidson and MacKinnon (1993) for an overview of this problem.

Since Bayesian methods of inference are unaffected by the possibility that the data generating process might be non-stationary, many Bayesian econometricians question the relevance of much of

[7] Extensions include time-varying transition probabilities (Filardo (1994), Durland and McCurdy (1994), Ghysels (1994)) as well as expanding the number of possible states (Sichel (1994)).

[8] Potter (1994), Koop and Potter (1994)

this literature - see, for example, Christiano and Eichenbaum (1990), Sims (1992) as well as the survey by Uhlig (1994). The major challenge posed by non-stationary data appears to be establishing "non-informative" priors; this point is discussed at length in Phillips (1991). Although the stochastic trends-deterministic trends choice poses its own specific complications, the problem of estimation and inference using diffuse priors has a long history - see, for example, Lindley (1957) as well as Chapters 5 and 6 in Bernardo and Smith (1994).

Since the parameters of a time series model rarely have an intuitive economic interpretation, eliciting a proper prior distribution for the parameters of the various models is not straightforward. In an analysis using proper priors, Koop (1992) uses Zellner's (1986) g-prior[9], and finds that inferences about the type of trend observed in output data are quite sensitive to the degree of *a priori* precision used in the analysis. This paper adopts the approach suggested by Atkinson (1978) and O'Hagan (1991), where proper priors are derived from an improper "proto-prior" and a "training sample" of observations prior to the sample. The role of this training sample is to provide the analyst with information about the parameters of the various models; inferences about the models themselves are done using data observed after the training sample.

3.2 Predictive Likelihoods and Model Evaluation

Since the focus of interest is the role of business cycle events in the choice of a model of long-run behaviour, Geweke's (1994) recommendation of the use of predictive odds ratios in model evaluation is particularly appealing. As we shall see, inspection of the individual predictive Bayes factors allows us to identify "important" observations that appear to play an inordinately large role in model evaluation.

Suppose that there are K models under consideration. Denote by M^m the hypothesis that model $m \in 1, 2, \ldots, K$ is valid, let $\theta^m \in \Theta^m$ represent the relevant parameter vector and let Y_t represent the cumulative information set available in period t. Given the model and the prior information set Y_t, prior beliefs about θ^m are characterised by $p(\theta^m|Y_t, M^m)$. Given Y_t and θ^m, the structure of the model provides us with the conditional density $p(y_{t+1}|Y_t, \theta^m, M^m)$. Now suppose that observe y_{t+1} is observed, so that the information set is now $Y_{t+1} = \{Y_t, y_{t+1}\}$. This new information is incorporated by applying Bayes' rule:

$$p(\theta^m|Y_{t+1}, M^m) = \frac{p(y_{t+1}|\theta^m, Y_t, M^m)p(\theta^m|Y_t, M^m)}{\int_{\theta^m \in \Theta^m} p(y_{t+1}|\theta^m, Y_t, M^m)p(\theta^m|Y_t, M^m)\ d\theta^m} \tag{7}$$

[9] The hyperparameter g represents prior precision as a proportion of that of the data.

Geweke (1994) suggests the use of predictive likelihoods in model comparison exercises. These are calculated by removing the dependance of $p(y_{t+1}|Y_t, \theta^m, M^m)$ on θ^m by integrating across the prior distribution $p(\theta^m|Y_t, M^m)$:

$$p(y_{t+1}|Y_t, M^m) = \int_{\theta^m \in \Theta^m} p(y_{t+1}|\theta^m, Y_t, M^m) p(\theta^m|Y_t, M^m)\ d\theta^m \tag{8}$$

Given a sample of τ new observations, the joint predictive likelihood for the sample $\{y_{t+j}\}_{j=1}^{\tau}$ can be calculated by exploiting the recursive structure of (7). Given a sequence of the predictive likelihoods in (8), the predictive density for the entire sample is simply the product of the individual predictive densities:

$$p(\{y_{t+j}\}_{j=1}^{\tau}|Y_t, M^m) = \prod_{j=1}^{\tau} p(y_{t+j}|Y_{t+j-1}, M^m) \tag{9}$$

The predictive likelihoods for each model can then be used to calculate the posterior probabilities associated with the various models:

$$p(M^k|\{y_{t+j}\}_{j=1}^{\tau}, Y_t) = \frac{p(\{y_{t+j}\}_{j=1}^{\tau}|Y_t, M^k)p(M^k|Y_t)}{\sum_{m=1}^{K} p(\{y_{t+j}\}_{j=1}^{\tau}|Y_t, M^m)p(M^m|Y_t)} \tag{10}$$

where $p(M^m|Y_t)$ is the prior probability associated with model m. Given a sequence of predictive likelihoods, the reader can make use of (10) to calculate posterior model probabilities for any combination of training sample, sample or prior model probabilities with the appropriate choice of t, j and $p(M^m|Y_t)$. Furthermore, the recursive nature of (10) can be exploited to identify the role of individual data points or of particular subsamples in the the transition between prior and posterior model probabilities.

Although it is possible to make use of these model probabilities in order to select a particular model, it may be difficult to justify discarding models with positive posterior probability. If the focus of the modelling exercise is to provide forecasts, then definitive statements about the appropriate time series model are not required; the optimal forecasting strategy would make use of all the available models, weighted by their posterior model probabilities.

3.3 Computing Predictive Likelihoods

In the models considered in this paper, there are no closed-form solutions for the integrals in (8). Instead, well-known techniques can be used to simulate artificial samples $\{\theta_j^m\}_{j=1}^{N}$ from the distributions $p(\theta^m|Y_t, M^m)$. Given these samples, the statistics $N^{-1}\sum_{j=1}^{N} p(y_{t+1}|\theta_j^m, Y_t, M^m)$ are consistent estimators for the predictive likelihoods. In principle, any desired degree of precision can be attained with the appropriate choice of N.

In the case of the linear models (1) and (2) and for the TAR models (5) and (6), sampling from $p(\theta^m|Y_t, M^m)$ is straightforward, since they can all be expressed as linear normal regression models. We adopt the reference proto-prior $p(\theta) \propto I_\rho \sigma^{-2}$, where I_ρ is an indicator variable equal to 1 if the parameters in $\rho(L)$ are consistent with the stationarity restrictions imposed by the models. Given this proto-prior and the training sample Y_t, the prior distribution is of the normal-gamma form, with hyperparameters taking the form of the least-squares estimates for the slope and variance parameters (Zellner (1971), pp 66-67). N *iid* draws can be simulated from this distribution, discarding draws that do not satisfy the stationarity conditions. In the results below, N is set equal to 5000.

Computing the predictive likelihoods for the Markov-switching models in (3) and (4) is more complicated, since the posterior distributions for the parameters of these models is non-standard. However, since the main econometric problem is posed by the fact that we do not observe the latent variable s_t, estimation is greatly simplified by exploiting Tanner and Wong's (1987) data augmentation technique and applying the Gibbs sampler. In order to keep attention focussed on the question of deterministic and stochastic trends, only a brief description of the estimation algorithm is provided; detailed treatments are provided by Albert and Chib (1993) and McCulloch and Tsay (1994). The sampling algorithm iterates through the sequence described below:

1. Given the data, observations for s_t as well as σ^2, the models (3) and (4) are linear normal regression models with known variance; the full conditional distribution for the vectors of the slope and intercept coefficents is multivariate normal with mean and covariance equal to the least-squares estimates. Values for these parameters can be easily simulated; draws that do not satisfy the stationarity conditions are discarded.

2. Given the data, observations for s_t as well as the slope and intercept coefficients, values for ε_t can be retrieved directly. Given these values and the reference prior, values for σ^2 are simulated from the inverse-gamma full conditional posterior.

3. Given s_0 and the T observations for s_t, calculate n_1^1, the number of times that $s_{t-1} = s_t = 1$ is observed, as well as n_0^0, the number of times that $s_{t-1} = s_t = 0$ is observed. Let T_1 and represent the number of observations where $s_{t-1} = 1$, and denote $T_0 = T - T_1$, the number of periods in which $s_{t-1} = 0$. Values for a and b are simulated from the full conditional beta distributions $\beta(37 + n_0^0, 13 + T_0 - n_0^0)$ and $\beta(45 + n_1^1, 5 + T_1 - n_1^1)$, respectively.

4. Given the data, the fixed parameters as well as well as the values of s_{t-1} and s_{t+1} apply Bayes' rule to the models (3) and (4) to compute the full conditional probability that $s_t = 1$. Values for

s_t can then be simulated recursively from a sequence of Bernoulli distributions. If the simulated sequence does not contain at least one turning point, the process is repeated until it does. This data augmentation step is the key element of the estimation algorithm; for more detail see Albert and Chib (1993) or McCulloch and Tsay (1994).

5. Given a, b and s_1, the probability that $s_0 = 1$ can also be computed by applying Bayes' rule to the conditional probabilities defined by the Markov process governing s_1; the value of s_0 can be simulated from the appropriate Bernoulli distribution.

It can be shown (Gelfand and Smith (1990), Casella and George (1992)) that the sequence generated by this algorithm forms an irreducible and aperiodic Markov chain that converges in distribution to $p(\theta^m|Y_t, M^m)$. After the sequence has converged, the sample generated by the subsequent draws is used to estimate the predictive likelihoods. In calculating the results presented below, the algorithm was initialized using 500 "burn-in" draws; the subsequent 5000 draws were used to compute the estimated predictive likelihoods.

4 Results

4.1 Data

The output series used here is US quarterly Gross Domestic Product (GDP) measured in constant 1987 dollars. As mentioned above, the data used in the training sample are for the period 1948-1951. Predictive likelihoods were calculated for the 171 observations between 1952:1 and 1994:3. For each of the six models, the number of lags is set equal to 4. This is the lag length used in Hamilton (1989); rounds of estimation of the linear model using values of r ranging from 2 to 6 generated results that are qualitatively similar to those presented below.

4.2 Prior Specification

The current application treats the model pairs $\{(1), (2)\}$, $\{(3), (4)\}$ and $\{(5), (6)\}$ as though they were non-nested hypotheses whose parameters are model-specific[10]. In specifying the proto-prior, it is noted that a maintained hypothesis for each model is that the autoregressive process used to characterise short-run fluctuations is stationary; long run-behaviour is presumed to be entirely described by the relevant model for the trend. Since this is the case, the paper adopts a proto-prior that is uniform

[10] It is possible to construct models that nest both the stochastic trends and deterministic trends models as special cases; this is the approach taken by - among others - DeJong and Whiteman (1991). In this case, interest revolves around whether the parameter restrictions associated with the various models are consistent with the data.

across the region of values for parameters in $\rho(L)$ such that the autoregressive process is stationary. Given that this region is compact, this "non-informative" prior is in fact proper. Standard reference priors are adopted for the other parameters in the various models.

Proper priors are also used for the transition probability parameters a and b; previous experience with Markov-switching models[11] indicates that non-informative priors for the transition probabilities yield implausible results. It appears that since recessionary periods are infrequent and of short duration, there is simply not enough information in the data to make the use of non-sample information irrelevant. McCulloch and Tsay (1994) find that the prior distributions $p(a) = \beta(37, 13)$ and $p(b) = \beta(45, 5)$ provide plausible results. The implied prior means for the durations of the phases are consistent with the belief that expansions last roughly 12 quarters, while recessions are generally only 4 quarters long. The standard deviations associated with these means are around 7 quarters and one quarter, respectively. Since these values appear to be reasonable, they are also used in this study. The NBER classifies 1947 as an expansionary period, so the prior probability $p(s_0 = 1)$ was set at 0.9 in step 5 of the Gibbs sampling algorithm described above.

We now turn to the choice for the length of the training sample. In order to ensure that inferences are based primarily on the data, the training sample should be fairly small. Berger and Pericchi (1992) suggest a procedure in which the analysis is performed with all combinations of training samples that assure proper priors (or a representative subset); the "intrinsic Bayes factor" is calculated by averaging across the Bayes factors generated by this set of priors. Although this approach is intriguing, the computational burden required to estimate the predictive likelihoods for many sets of priors appears to be too great to apply it to the problem addressed in this paper. In any case, it may be difficult to interpret the notion of a training sample for a time series model that contains observations that occur *after* the data.

In choosing the size for the training sample, an extra complication is posed by the two non-linear models. In the case of the Markov-switching model, if the series s_t does not contain at least one turning point, it would be perfectly correlated with the intercept. Although the estimation algorithm makes sure that the simulated series s_t always contains at least one turning point, estimating the Markov-switching model only makes sense if it is believed that the data do in fact contain a turning point. This is a fairly innocuous assumption for the entire data set, but it may not be reasonable to assume that any small subset of the data will also contain a turning point. Similarly, if the data do not

[11] Albert and Chib (1993), McCulloch and Tsay (1994), Filardo and Gordon (1993)

include at least one observation where CDR_t is not equal to zero, the TAR model will be unidentified.

Taking these factors into consideration, It was decided that a training sample using output data during the four-year period 1948-1951 is arguably the smallest[12] plausible training sample. The NBER identifies two turning points during this period: a peak in November 1948, as well as a trough in October of 1949. The existence of two turning points makes estimation of the Markov-switching and the TAR models feasible using data from the training sample alone, so prior beliefs about the business cycle features of the data will be based on data that contain business cycle events. The small size of the training sample means that the information contained in the priors will be relatively weak in comparison with the more than forty years of data used to evaluate the competing models.

Although the prior incorporates the information provided by a complete recessionary phase, it does not contain an example of an entire expansionary phase; the training sample truncates the expansions adjoining the 1948-9 recession. If it were felt that the training sample should include an entire expansion as well as a complete recession, then a longer training sample would be appropriate. The NBER identifies the next peak as occurring in July of 1953, so another plausible training sample might be the period 1948-53. Others may find that an appropriate training sample should be even longer. Robustness of conclusions with respect to the choice of the training sample is easily verified, since the individual predictitive likelihoods are not affected by the choice of the size of the training sample. Readers who wish to use a longer training sample need only concern themselves with the predictive likelihoods that are associated with data points after the end of their preferred training sample.

4.3 Estimation Results

The logs of the predictive likelihoods for each of the six models considered are presented in the Appendix. It appears that 5000 draws is enough to estimate the predictive likelihoods with a fairly high degree of accuracy. In the case of the linear and for the TAR models, evaluating the precision for the mean of 5000 *iid* draws is straightforward; the estimated standard error is typically around 0.5% of its estimate. In the case of the Markov-switching model, visual inspection suggests that the sequence of simulated predictive likelihoods converges well before the 500^{th} iteration, and the numerical standard errors for the mean of the subsequent 5000 draws - calculated using Geweke's (1992) method - are of the same order as those calculated for the other models. Estimation of the the linear and TAR models

[12] The data for 1947 are used in constructing the right-hand-side variables of the regression equations. Note also that between 5 and 7 degrees of freedom are lost in assuring that the prior is proper.

was fairly rapid - the average time[13] to compute the predictive likelihood for a given data point was roughly 2 minutes. Estimation time for the Markov-switching models (3) and (4) was much slower; the computational burden of the data augmentation step slowed the estimation time to around 10 minutes.

Although the parameters of the models are not the direct focus of the study, it may be of interest to characterise the information about the parameters contained in the priors. In a preliminary round of estimation, the models were estimated using the proto-priors and the training sample described above in order to provide estimates for the prior moments. These estimates are reported in the Appendix. The prior standard deviations for each of the parameters (except the transition probability parameters, for which tight proto-priors are adopted) are all fairly large, suggesting that although the prior is proper, it is also fairly diffuse. It should also be noted that although the prior means for the parameters of the deterministic models are not far from the case in which output follows a random walk with drift, the prior only has mass in the region in which the output series is trend-stationary. The tables also report the posterior moments calculated from the main data set. The posterior means are generally similar to the prior means, although they are more precisely estimated.

4.4 Model Evaluation

The log predictive likelihoods for the entire sample - which are simply the sums of the individual log predictive likelihoods listed in the Appendix - are presented in Table 1. ST and DT represent models that make use of stochastic trends and deterministic trends, respectively, and LIN, MSW and TAR represent the models that use linear, Markov-switching and Threshold Autoregressive representations for the stationary component of output.

Table 1: Estimated Logs of Predictive Likelihoods

	LIN	MSW	TAR
ST	552.23	551.89	553.28
DT	551.66	555.83	551.87

The log predictive likelihoods in Table 1 can be used in (10) to calculate the posterior probabilities of the hypotheses of interest. For example, it has been noted that empirical investigations into whether or not output has a stochastic trend usually restrict attention linear models of the business cycle. In

[13] All computations were done in MATLAB using a Sun SPARCstation 5.

the notation of this paper, this is equivalent to calculating the posterior probability $p(\mathrm{ST}|\mathrm{LIN}, Y_T)$, where the set of models under consideration is limited to {ST-LIN,DT-LIN}. If the prior probabilities for ST and DT are both set equal to 0.5[14], then $p(\mathrm{ST}|\mathrm{LIN}, Y_T)$ can be expressed as:

$$p(\mathrm{ST}|\mathrm{LIN}, Y_T) = \frac{\exp[552.23] \cdot 0.5}{(\exp[552.23] \cdot 0.5) + (\exp[551.66] \cdot 0.5)} = 0.638$$

The slight upward revision in the probability associated with the stochastic trends hypothesis is roughly consistent with the results found in the empirical literature - the ST hypothesis is favoured, but the evidence is weak.

We now turn to the role played by the use of the linear model in tests about long-run output patterns. Table 2 provides the posterior probabilities for the stochastic trends hypothesis for given each of the three business cycle models considered; in each case, equal prior weight is given to the ST and DT hypotheses. It is clear from Table 2 that the support for the stochastic trends hypothesis varies greatly according to the model for the business cycle: the ST hypothesis receives almost no support if the Markov-switching model is used, and it receives strong support if the TAR model is used to capture the stationary dynamics of output. If no reason is given for preferring one of the business cycle models, no definitive conclusions about the nature of the trend in US GDP can be drawn from Table 2.

Table 2: Conditional Posterior Probabilities for Stochastic Trends

$p(\mathrm{ST}|\mathrm{LIN}, Y_T)$	0.638
$p(\mathrm{ST}|\mathrm{MSW}, Y_T)$	0.019
$p(\mathrm{ST}|\mathrm{TAR}, Y_T)$	0.804

If the attention of the analyst is addressed to the stationary component of output, the relative merits of the LIN, MSW and TAR models are of interest. Indeed, even if the focus of interest is limited to comparing the ST and DT hypotheses, the results of Table 2 suggest that some attention should be paid to this question. Table 3 provides the posterior probabilities for the business cycle models, conditioned on both the ST and DT hypotheses.

Table 3 illustrates how inferences about business cycle models depend on the way in which the non-stationary component of output is characterised. If a stochastic trend is imposed, the linear

[14]Equal prior weight is given to each model in each of the model comparison exercises presented below.

Table 3: Conditional Posterior Probabilities for Business Cycle Models

H_i	$p(H_i \mid \mathrm{ST}, Y_T)$	$p(H_i \mid \mathrm{DT}, Y_T)$
LIN	0.219	0.015
MSW	0.156	0.967
TAR	0.625	0.018

specification is favoured over the Markov-switching model, and the TAR model is preferred over the linear specification - these findings are consistent with the conclusions reached by Hansen (1992)[15] and by Beaudry and Koop (1993). However, if a deterministic trend is imposed, then the Markov-switching model is strongly favoured over the other two specifications. As was the case in Table 2, no conclusions about the validity of the various business cycle models can be drawn from Table 3 if the nature of the trend is unknown.

Tabels 2 and 3 also suggest that iterating between tests for trends and tests for business cycle models is unlikely to generate definitive conclusions. If a linear model is used to choose a specification for the trend, Table 2 suggests that the ST representaion will be chosen. Given the ST hypothesis, Table 3 indicates that the TAR model will be preferred. Since the ST model was chosen using the linear model, Table 2 can then be used again to see if using the TAR model affects the choice for the trend specification: since $p(\mathrm{ST} \mid \mathrm{TAR}, Y_T) = 0.804$, the choice of the ST-TAR combination would appear to be confirmed. However, if the original test for the trend had made use of the Markov-switching model, Table 2 indicates that a deterministic trend should be used. Moreoever, if the DT specification is used to choose a business cycle model, Table 3 would confirm the original choice of the Markov-switching specification. An iterative testing procedure would end up at either the ST-TAR or the DT-MSW model, depending on the auxiliary hypothesis used in the first step.

Clearly, more information is needed than that contained in the conditional model probabilities reported in Tables 2 and 3. Table 4 reports the joint probabilities for each of the six model pairs generated by combining the two models for the trend with the three business cycle models. These joint probabilities can then be used to calculate the marginal probabilities for the hypotheses of interest.

If the focus of interest is in comparing the ST and DT hypotheses, Table 4 indicates that by integrating across the uncertainty associated with each of the three auxiliary hypotheses for the business cycle,

[15] Note that the theoretical difficulties associated with classical evaluation of the Markov-switching model are not encountered in a Bayesian model comparison exercise.

Table 4: Joint and Marginal Model Probabilities

	LIN	MSW	TAR	
ST	0.023	0.017	0.067	0.107
DT	0.013	0.863	0.016	0.893
	0.037	0.880	0.083	

we obtain $p(\text{ST}|Y_T) = p(\text{ST,LIN}|Y_T)+p(\text{ST,MSW}|Y_T)+p(\text{ST,TAR}|Y_T) = 0.023+0.017+0.067 = 0.107$. Although the stochastic trends model is favoured if either the linear or the TAR models are used to describe the stationary component of output data, the marginal probabilities associated with these models is small - 0.037 and 0.083, respectively. Of the six models considered, the Markov-switching specification with a deterministic trend receives enough support such that the marginal probabilities for both the DT and MSW hypotheses are higher than the marginal probabilities for the alternative hypotheses.

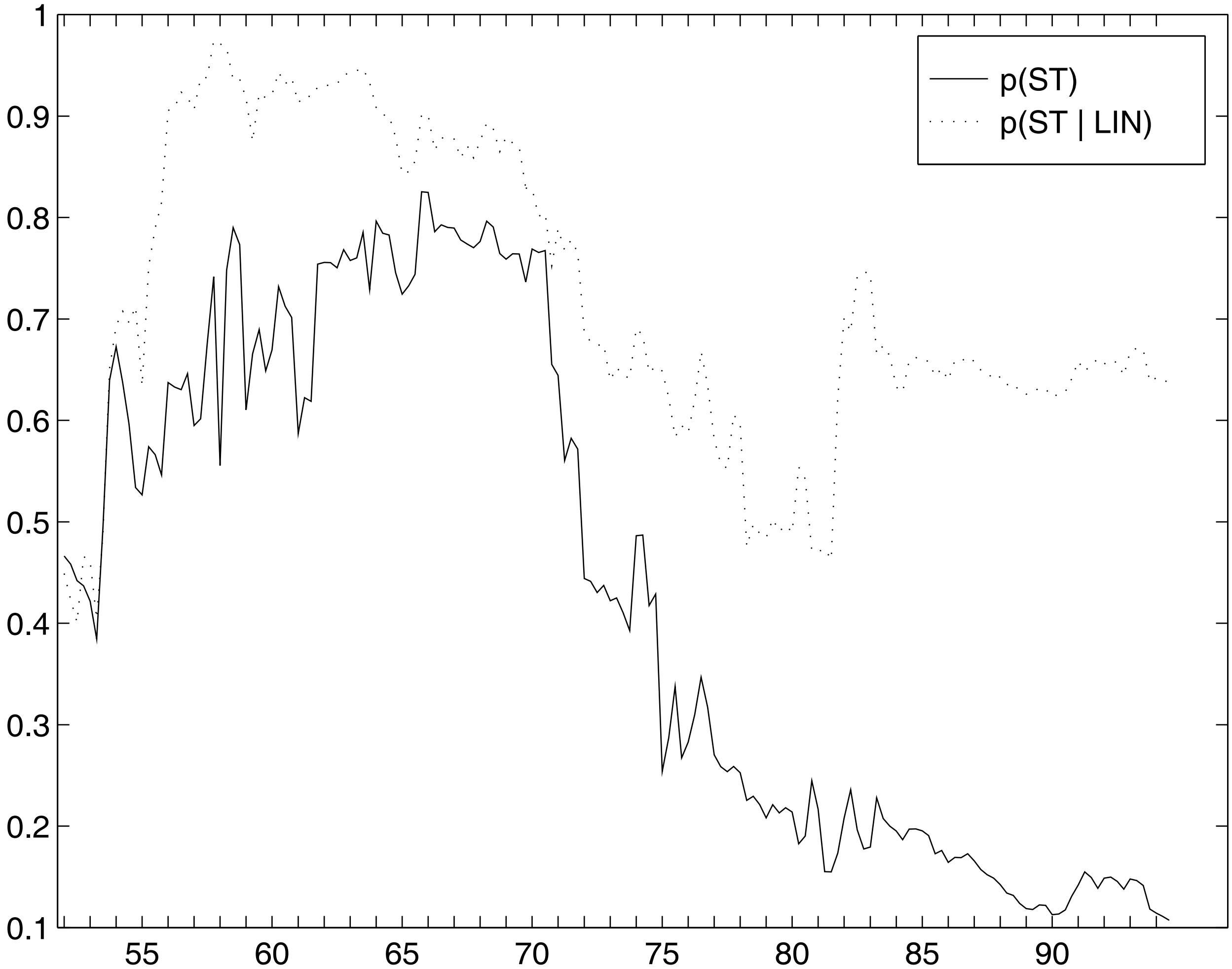
4.5 Sub-sample Variations in Model Support

An intriguing feature of the use of the individual predictive likelihoods is that it is possible identify variations in the degree to which the data support the various models throughout the sample. By repeated application of (10), it is possible to calculate the sequence of updated posterior model probabilities associated with the accumulated data set available at each observation. Figures 1-3 illustrate that the support for the various hypotheses of interest is far from evenly distribued throughout the sample.

Figure 1 plots the sequence of updated probabilities for the stochastic trends hypothesis. The solid line graphs $p(\text{ST}|Y_t)$ for each observation, while the dotted line traces the sequence $p(\text{ST}|\text{LIN}, Y_t)$. Although the accumulated evidence of the entire data set favours the adoption of the deterministic trends hypothesis, it is clear from Figure 1 that the support for the DT model varies greatly over the sample. During the 75 observations, the probability for the ST model is generally revised upwards and reaches 0.767 by the end of 70:3. However, the subsequent 18 observations contain enough support for the DT hypothesis to sharply revise the posterior probability for ST down to 0.254 by 75:1. The subsequent 78 data points also tend to favour the DT model, but to a much more modest extent.

As mentioned earlier, most investigations of the trend in output limit attention to linear models of the business cycle; Figure 1 also illustrates the sample variation in the support for the ST case

Figure 1: Cumulative Posterior Probabilities for the Stochastic Trends Model



conditional on the use of linear model. For the first part of the sample, there appears to be no loss in restricting attention to the linear model; the ST model receives the same support from the data observed up to 1970:3 whether or not non-linear business cycle models are taken into account. The data observed in the 1970's generally favour the DT model, although restricting attention to linear models attenuates the downward revision for the probability of the ST hypothesis; the probability of the ST model is reduced to 0.463 by 1981:3. The 1981:4 observation plays a remarkably important role: the probability for the ST case jumps to 0.623. The subsequent 51 data points generate no significant revision in the probabilities associated with the ST model. Indeed, the sharp jump in 1981:4 has the effect of compensating for the loss of support for the ST hypothesis over the period 1970:4-1981:3, so the data over the entire period 1970:4-1994:3 have almost no effect on the probability associated with the stochasic trends hypothesis.

Figure 2: Cumulative Posterior Probabilities for Business Cycle Models

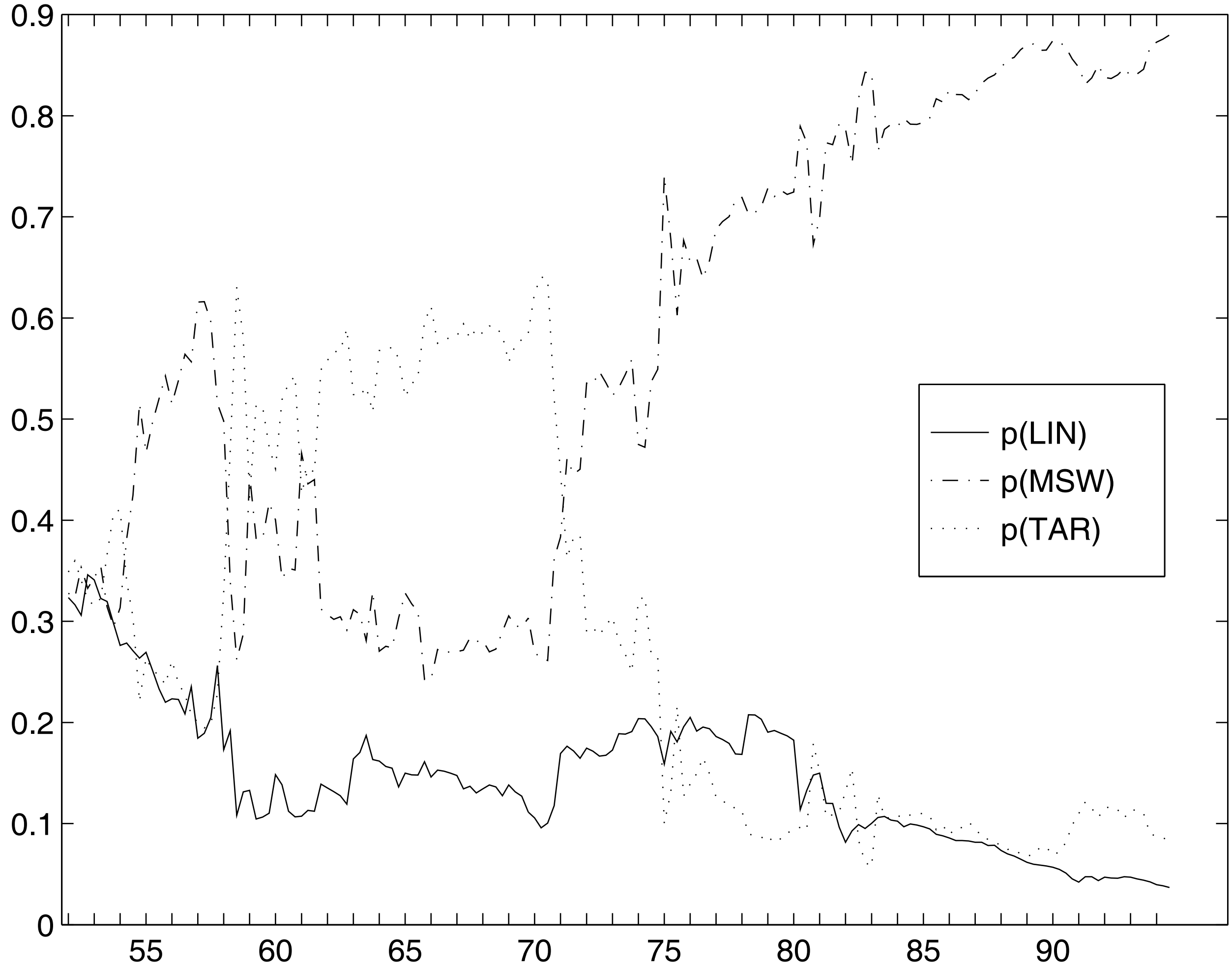
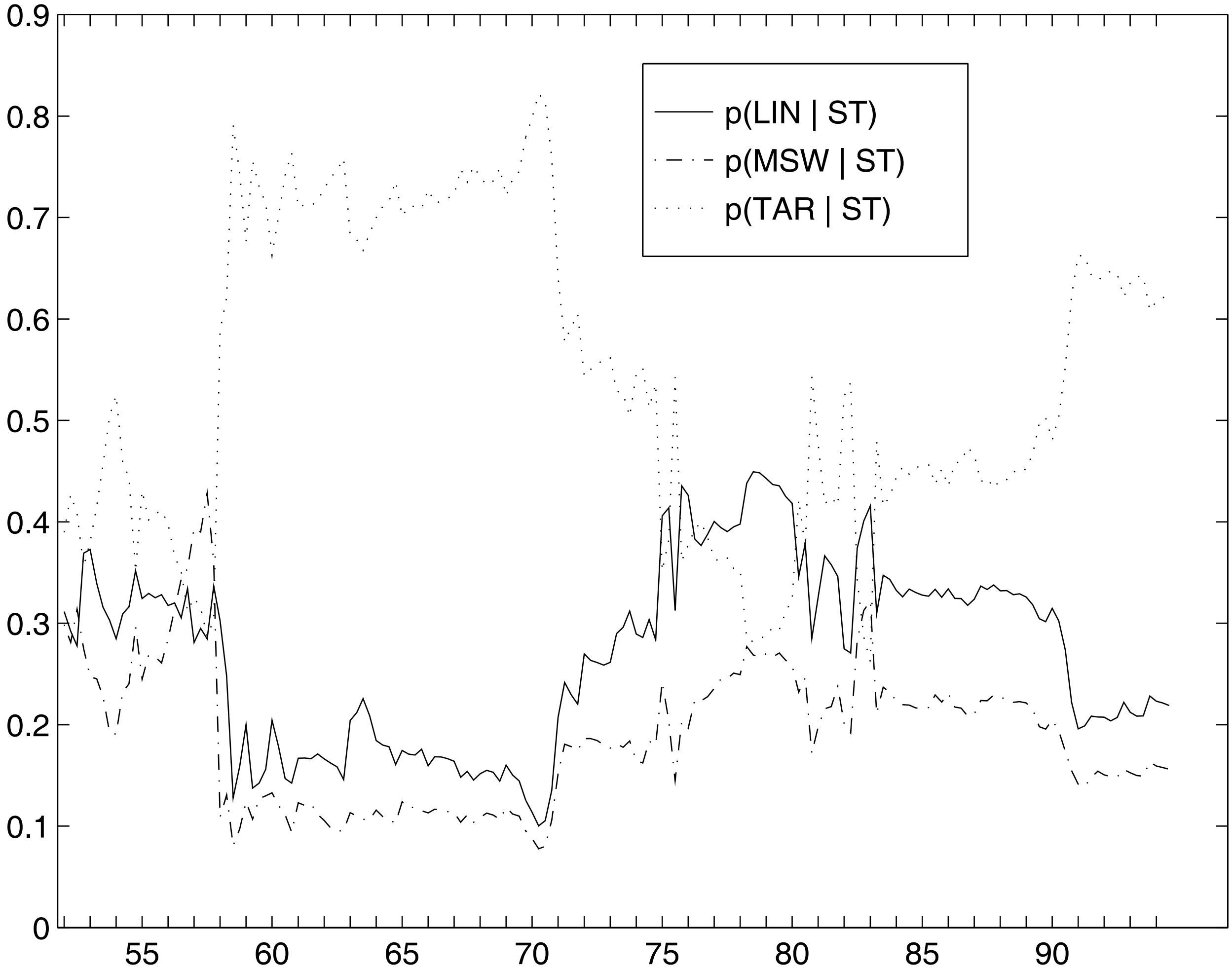


Figure 2 traces the sequence of updated probabilities for each of the three business cycle specifications considered. As was the case in Figure 1, the support for the various models varies a great deal across the sample. The first few years of data generally favour the Markov-switching model, but the probability associated with the MSW model falls from 0.616 to 0.262 over the period 1957:2-1958:3, while the support for the TAR model increases from 0.195 to 0.630 over the same period. The probabilities for the LIN, MSW and TAR models fluctuate slightly over the subsequent 48 observations, but the model probabilities in 1970:3 are roughly the same as those in 1958:3. After 1970:3, the ranking of the MSW and TAR model is revised sharply, with model probabilities reaching 0.741 and 0.100, respectively by 1975:1. The probability associated with the Markov-switching model is generally revised upward throughout the the rest of the sample.

Figure 3 illustrates the evolution of the support for the business cycle models if a stochastic trend is

Figure 3: Cumulative Posterior Probabilities for Business Cycle Models: ST Case



used to represent the non-stationary component of US GDP. Although there are obvious differences in scale, Figure 3 shares some of the basic features of Figure 2 over the first part of the sample, including a sharp increase in the support for the TAR model at the expense of the MSW model over the period 1957-58, followed by a period of only minor revisions in the model probabilities. As in Figure 2, the data observed after 1970:3 have the effect of revising the TAR model probabilities sharply downwards. The principle difference between Figures 2 and 3 is that the probabilities for the Markov-switching model are revised only slightly; the linear model is the main beneficiary of the loss of support for the TAR specification and by 1975:1, the linear specification has the most posterior support. The recessionary periods of the early 1980's generate no fewer than three changes in the ordering for the TAR and linear models, but for the period 1983:2-1990:2, there are no significant revisions for the various model probabilities. The arrival of the recession in 1990:3 has the effect of increasing the support for the TAR specification at the expense of both the MSW and linear models, so that the posterior probability for the TAR model is roughly the same as it was in 1970:3. No further revisions occur over the last three years of the sample.

Figures 1-3 suggest that for the data observed up to 1970:3, the use of the auxilary hypotheses *i* and *ii* described in the Introduction do not appear to greatly distort inferences neither about the trend in US GDP nor about the most appropriate way to model the stationary dynamics of output. The data up to 1970:3 generally support the stochastic trends hypothesis as well as the TAR model regardless of the way in which the auxiliary hypotheses are specified.

This is not the case for data observed after 1970:3. If a linear model is used to describe the stationary dynamics of output, the data over the period 1970:4-1994:3 are unable to distinguish between the ST and DT hypotheses. On the other hand, if non-linear business cycle models are considered, then the observations between 1970:4 and 1994:3 have the effect of reducing the probability for the stochastic trends hypothesis from 0.804 to 0.107. Similarly, if a stochastic trend is imposed, the 1970:4-1990:3 data do not significantly revise the posterior probability for the TAR model, but if both stochastic and determinist trends are taken into account, the data observed during this paper has the effect of revising the probability for the Markov-switching model sharply upward, at the expense of both the TAR and linear models.

4.6 Trends, Cycles and Turning Points

It has already been noted that inferences about the nature of the trend in US GDP depend on the form for the *model* for the business cycle. An additional conclusion that can be drawn from Figure 1 is that inferences about the trend apear to be quite sensitive to the occurrence of business cycle *events*. An important feature of Figure 1 is that movements in the cumulative probability for the stochastic trends hypothesis are from from smooth; certain data points contain enough information to generate sharp movements in the cumulative probability for the stochastic trends hypothesis.

Inspection of the sequence $p(\mathrm{ST}|Y_t)$ reveals an intriguing pattern that is closely related to the business cycle. It is generally the case that the data points observed at the onset of a recession produce strong upward revisions in favour of the ST hypothesis, while the data points observed at the beginning of an expansion generate important downward revisions in the cumulative probability for the stochastic trends hypothesis. These effects are not symmetric, and their relative size varies over the sample. As Figure 1 illustrates, the upward revisions at business cycle peaks outweigh the effects of the downward revions at the troughs over the first part of the sample, while the reverse appears to be the case following the sharp drop in the probability for the ST hypothesis associated with the trough in late 1970. Towards the end of the sample, the relative importance of individual data points is reduced, thus attenuating the absolute size of the revisions associated with the turning points.

Less surprisingly, the cumulative probabilities for the various business cycle models also display sharp jumps at the turning points, since these models are distinguished by the way in which they characterise business cycles. During expansionary phases, the predictions of all three models are essentially equivalent, so the data observed during these periods - which comprise the greater part of the sample - are generally unable to distinguish between the three models. The predictions of the models differ most at the turning points, so the observations at the peaks and the troughs of the cycle provide most of the information about the relative merits of the business cycle models.

Clearly, the information contained in the data is not uniformly distributed across the sample; information about the appropriate model for both the stationary and non-stationary component of US GDP appears to be concentrated at data points that are generally associated with business cycle turning points. Since turning points are relatively rare (the NBER identifies 16 over the period 1952-1994), it is perhaps unsurprising that it has proven to be difficult to make definitive conclusions about the nature of the trend in output.

In the Bayesian framework, strongly informative data will generally generate inferences that are robust the the specification of prior beliefs. Koop's (1992) finding that inferences about the trend in output are highly sensitive to the level of prior precision suggests that the information contained in the data is in fact quite modest.

5 Conclusion

The main finding of the paper is that there is a strong relationship between inferences about trends, inferences about business cycle models, and the data points associated with business cycle turning points. Inferences about the nature of the trend are *not* robust to variations in the specification for the business cycle movements in output, and conclusions about the appropriate model for the business cycle depend on the specification for the non-stationary dynamics of output. Moreover, the available information about both the trend and the cycle appears to be concentrated at business cycle turning points.

Although the empirical results presented above favour the use of a determinstic trend for the non-stationary component of US GDP and a Markov-switching model for the business cycle dynamics of output, these conclusions are based on a very small subset of the available models for both the trend and the cycle. A comprehensive search across all model pairs would be a tedious exercise, and since the information in the available data set appears to be fairly weak, it is unlikely that such a specification search would yield strong results.

If there were compelling economic reasons for doing so, then the computational burden of a more comprehensive model comparison exercise might be justified. Typically, studies of the non-stationary component of output are motivated by a desire to see whether or not the 'observed' trend is consistent with that predicted by a particular theory of growth. Similarly, studies of the staionary component of output are interested in identifying features of the data that should be explained by a well-specified theory of the business cycle. Since these are important issues, it is hardly surprising that a great deal of effort has gone into trying to identify the appropriate way of identifying the trend and the cycle. However, from an economic point of view, it is far from clear that the trend-cycle distinction is of sufficient importance to justify the effort that has gone into empirical investigations of the question. In his survey of real business cycle models, Plosser (1989) quotes Hicks' (1965) comment on the link between the economics and the econometrics of the trend-cycle distinction:

> The distinction between trend and fluctuation is a statistical distinction; it is an unquestionably useful device for statistical summarizing. Since economic theory is to be applied to statistics, which are arranged in this manner, a corresponding arrangement of theory will (no doubt) often be convenient. But this gives us no reason to suppose that there is anything corresponding to it on the economic side which is at all fundamental. We have no right to conclude, from the mere existence of the statistical device, that the economic forces making for trend and for fluctuation are any different, so that they have to be analyzed in different ways.

Faced with the difficulties involved in making definitive conclusions about the 'true' nature of the trend or about the 'true' business cycle, it is not clear that the economic importance of the question justifies attempting to provide an answer.

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Appendix

Prior and Posterior Moments

Table 5: Prior and Posterior Moments - Linear Models

	Stochastic Trend		Deterministic Trend	
Parameter	Prior	Posterior	Prior	Posterior
α			2.0388 (1.48)	-0.2960 (0.17)
β	0.0090 (0.01)	0.0050 (0.001)	0.0197 (0.04)	0.0069 (0.002)
ρ_1	0.3192 (0.30)	0.3361 (0.07)	1.0018 (0.30)	1.2980 (0.07)
ρ_2	0.2396 (0.34)	0.1295 (0.08)	-0.0383 (0.46)	-0.2003 (0.12)
ρ_3	0.1028 (0.30)	-0.0535 (0.08)	-0.0736 (0.43)	-0.1927 (0.12)
ρ_4	-0.3150 (0.28)	-0.0706 (0.08)	-0.1781 (0.30)	0.0596 (0.07)
σ	0.0163 (0.004)	0.0094 (0.001)	0.0144 (0.004)	0.0092 (0.001)

Table 6: Prior and Posterior Moments - Markov Models

	Stochastic Trend		Deterministic Trend	
Parameter	Prior	Posterior	Prior	Posterior
α_0			2.0157 (0.44)	-0.5662 (0.13)
α_1			0.0245 (0.01)	0.0111 (0.004)
β			0.0141 (0.02)	0.0061 (0.001)
β_0	0.0025 (0.01)	0.0024 (0.003)		
β_1	0.0256 (0.01)	0.0057 (0.003)		
ρ_1	0.0331 (0.32)	0.2799 (0.09)	0.7669 (0.27)	1.1989 (0.09)
ρ_2	0.1097 (0.29)	0.1085 (0.08)	0.0434 (0.38)	-0.1404 (0.12)
ρ_3	0.1759 (0.35)	-0.0666 (0.08)	0.0772 (0.35)	-0.1669 (0.12)
ρ_4	-0.2133 (0.31)	-0.0790 (0.08)	-0.2177 (0.24)	0.0830 (0.05)
a	0.8786 (0.05)	0.8677 (0.05)	0.8729 (0.05)	0.8304 (0.05)
b	0.8917 (0.04)	0.8972 (0.04)	0.8980 (0.04)	0.9261 (0.03)
σ	0.0115 (0.005)	0.0090 (0.001)	0.0101 (0.004)	0.0083 (0.001)

Table 7: Prior and Posterior Moments - TAR Models

	Stochastic Trend		Deterministic Trend	
Parameter	Prior	Posterior	Prior	Posterior
α_0			2.2401 (1.72)	-0.1239 (0.18)
α_1			0.4575 (1.61)	-0.2942 (0.12)
β			0.0200 (0.05)	0.0070 (0.002)
β_0	0.0028 (0.009)	0.0018 (0.002)		
β_1	-1.3774 (1.30)	-0.3419 (0.12)		
ρ_1	0.5055 (0.34)	0.4359 (0.08)	0.9037 (0.45)	1.3904 (0.08)
ρ_2	0.2549 (0.33)	0.2090 (0.08)	-0.0182 (0.52)	-0.2176 (0.12)
ρ_3	0.0828 (0.30)	-0.0065 (0.08)	-0.0197 (0.43)	-0.2116 (0.12)
ρ_4	-0.2899 (0.27)	-0.0294 (0.07)	-0.1901 (0.30)	0.0076 (0.08)
σ	0.0155 (0.004)	0.0092 (0.001)	0.0153 (0.004)	0.0091 (0.001)

Estimated Log Predictive Likelihoods

NOTES: NBER = NBER reference cycle
NBER = 1 for expansions
NBER = 0 for recessions
Δy_t = per cent growth rate of GDP

			Linear Models		Markov Models		TAR Models	
Period	NBER	Δy_t	ST	DT	ST	DT	ST	DT
52:1	1	0.73	2.91	3.12	2.87	3.17	3.14	3.05
52:2	1	0.27	3.08	3.19	3.09	3.20	3.24	3.15
52:3	1	0.61	3.15	3.24	3.31	3.33	3.16	3.21
52:4	1	2.19	2.97	2.71	2.56	2.69	2.55	2.74
53:1	1	1.76	3.27	3.29	3.15	3.41	3.33	3.23
53:2	1	0.47	2.66	2.89	2.74	2.93	2.84	2.90
53:3	0	-0.44	2.67	2.32	2.66	2.23	2.83	2.33
53:4	0	-0.99	3.08	2.42	2.96	2.71	3.22	2.37
54:1	0	-0.62	3.24	3.05	3.29	3.33	3.34	2.86
54:2	0	-0.14	3.27	3.20	3.38	3.47	3.05	3.05
54:3	1	1.16	3.29	3.35	3.31	3.52	3.24	3.17
54:4	1	1.22	3.37	3.29	3.47	3.61	3.03	3.19
55:1	1	2.57	2.98	3.34	2.87	3.02	3.27	3.18
55:2	1	0.92	3.01	2.46	3.09	2.93	2.93	2.34
55:3	1	1.11	3.41	3.20	3.42	3.51	3.44	3.16
55:4	1	0.88	3.46	3.31	3.43	3.57	3.46	3.29
56:1	1	-0.39	3.24	2.46	3.36	2.94	3.24	2.60
56:2	1	0.64	3.49	3.46	3.58	3.51	3.40	3.44
56:3	1	0.21	3.50	3.30	3.63	3.57	3.50	3.37
56:4	1	1.34	3.39	3.46	3.33	3.21	3.18	3.36
57:1	1	0.54	3.10	3.24	3.38	3.51	3.32	3.33
57:2	1	-0.09	3.27	2.83	3.21	3.21	3.19	2.97
57:3	1	0.89	3.51	3.50	3.64	3.21	3.45	3.45
57:4	0	-1.12	2.65	1.73	2.31	2.18	2.54	1.96
58:1	0	-2.23	1.19	1.31	0.10	2.15	1.96	0.90
58:2	1	0.80	2.42	2.56	2.80	1.73	2.67	3.02
58:3	1	2.00	2.28	2.99	2.46	2.69	3.19	3.27
58:4	1	2.15	3.36	3.35	3.33	3.23	3.08	3.37
59:1	1	1.27	3.00	3.27	3.01	3.56	2.67	3.39
59:2	1	1.70	2.94	3.41	3.17	3.06	3.43	3.36
59:3	1	-0.35	2.89	2.36	3.03	2.76	2.82	2.60
59:4	1	0.57	3.40	3.46	3.33	3.50	3.29	3.45
60:1	1	1.79	3.08	3.04	2.83	2.71	2.74	2.59
60:2	0	-0.26	2.81	2.46	2.85	2.65	3.00	2.67
60:3	0	0.10	3.23	3.43	3.34	3.52	3.48	3.49
60:4	0	-0.64	3.11	3.01	2.97	3.19	3.17	3.25
61:1	1	0.83	3.03	3.39	3.14	3.38	2.80	3.36
61:2	1	1.44	3.49	3.41	3.47	3.35	3.50	3.18
61:3	1	1.45	3.51	3.52	3.53	3.53	3.51	3.51
61:4	1	2.01	3.45	3.32	3.33	2.76	3.43	3.30
62:1	1	1.31	3.55	3.54	3.53	3.57	3.60	3.56
62:2	1	1.04	3.57	3.56	3.53	3.60	3.61	3.59
62:3	1	0.79	3.59	3.57	3.58	3.65	3.62	3.60
62:4	1	-0.11	3.40	3.26	3.49	3.39	3.50	3.35
63:1	1	1.42	3.17	3.11	3.00	2.88	2.73	2.92
63:2	1	1.31	3.58	3.55	3.52	3.53	3.54	3.53
63:3	1	1.72	3.49	3.44	3.39	3.27	3.41	3.43
63:4	1	0.75	3.28	3.53	3.36	3.68	3.39	3.53

			Linear Models		Markov Models		TAR Models	
Period	NBER	Δy_t	ST	DT	ST	DT	ST	DT
64:1	1	2.48	1.97	2.31	2.18	1.66	2.12	2.25
64:2	1	0.81	3.37	3.45	3.34	3.47	3.41	3.41
64:3	1	1.17	3.55	3.56	3.52	3.57	3.57	3.58
64:4	1	0.28	3.32	3.53	3.40	3.64	3.45	3.55
65:1	1	1.93	2.39	2.70	2.49	2.39	2.26	2.52
65:2	1	1.35	3.58	3.57	3.55	3.56	3.62	3.58
65:3	1	1.73	3.50	3.42	3.51	3.45	3.51	3.46
65:4	1	2.32	3.24	2.80	3.17	2.69	3.20	2.92
66:1	1	2.06	3.37	3.38	3.45	3.48	3.49	3.48
66:2	1	0.18	2.78	3.11	2.75	2.96	2.71	2.92
66:3	1	0.95	3.53	3.43	3.53	3.51	3.54	3.44
66:4	1	0.55	3.62	3.62	3.61	3.65	3.63	3.62
67:1	1	0.63	3.62	3.64	3.63	3.65	3.65	3.63
67:2	1	0.44	3.49	3.65	3.51	3.66	3.63	3.67
67:3	1	1.14	3.57	3.46	3.59	3.57	3.51	3.39
67:4	1	0.58	3.55	3.65	3.54	3.62	3.63	3.67
68:1	1	1.35	3.58	3.43	3.58	3.51	3.52	3.39
68:2	1	1.61	3.56	3.39	3.58	3.42	3.53	3.39
68:3	1	0.71	3.54	3.59	3.53	3.58	3.55	3.58
68:4	1	0.17	3.34	3.55	3.36	3.55	3.41	3.55
69:1	1	1.51	3.13	3.00	3.13	3.07	2.99	2.90
69:2	1	0.11	3.30	3.35	3.30	3.33	3.38	3.37
69:3	1	0.58	3.65	3.68	3.67	3.69	3.70	3.69
69:4	0	-0.33	3.02	3.34	3.02	3.31	3.21	3.40
70:1	0	-0.25	3.50	3.51	3.52	3.41	3.62	3.54
70:2	0	-0.29	3.37	3.55	3.38	3.51	3.53	3.54
70:3	1	1.23	3.33	3.31	3.31	3.26	3.27	3.41
70:4	1	-0.75	1.80	2.10	1.83	2.09	1.48	2.27
71:1	1	2.29	2.30	2.09	2.23	1.86	1.70	2.29
71:2	1	0.15	3.10	3.22	3.12	3.31	2.84	3.21
71:3	1	0.61	3.53	3.48	3.57	3.50	3.61	3.52
71:4	1	0.51	3.60	3.67	3.62	3.68	3.66	3.69
72:1	1	2.00	2.65	3.05	2.51	2.95	2.34	2.91
72:2	1	1.71	3.56	3.60	3.58	3.59	3.59	3.64
72:3	1	1.16	3.63	3.65	3.63	3.69	3.64	3.60
72:4	1	1.58	3.56	3.56	3.55	3.53	3.58	3.63
73:1	1	2.42	2.60	2.75	2.57	2.60	2.59	2.89
73:2	1	0.44	3.23	3.17	3.15	3.13	3.07	2.92
73:3	1	-0.10	3.14	3.18	3.11	3.20	3.12	3.09
73:4	1	0.76	3.56	3.57	3.54	3.58	3.46	3.56
74:1	0	-0.91	2.67	2.44	2.63	2.32	2.82	2.59
74:2	0	0.25	3.65	3.69	3.65	3.66	3.68	3.66
74:3	0	-0.88	2.69	2.86	2.76	2.94	2.56	2.74
74:4	0	-0.39	3.36	3.33	3.40	3.42	3.47	2.97
75:1	0	-2.27	0.32	0.34	0.26	0.85	-0.46	-0.68
75:2	1	1.15	2.80	2.91	2.61	2.56	2.86	3.32
75:3	1	1.85	2.84	3.00	2.78	2.85	3.47	3.41
75:4	1	1.30	3.51	3.46	3.51	3.53	2.77	3.34
76:1	1	1.93	3.55	3.58	3.54	3.48	3.62	3.57
76:2	1	0.37	3.15	3.02	3.39	3.15	3.30	3.02
76:3	1	0.35	3.58	3.38	3.60	3.44	3.62	3.44
76:4	1	1.04	3.47	3.62	3.46	3.57	3.40	3.55
77:1	1	1.46	3.39	3.61	3.39	3.58	3.30	3.54
77:2	1	1.68	3.53	3.63	3.58	3.60	3.54	3.60
77:3	1	1.39	3.67	3.69	3.69	3.71	3.69	3.70
77:4	1	-0.20	2.79	2.57	2.80	2.78	2.75	2.54
78:1	1	0.69	3.65	3.69	3.63	3.67	3.64	3.69
78:2	1	3.16	0.11	0.60	0.12	0.11	-0.20	0.22
78:3	1	0.77	3.30	3.22	3.25	3.26	3.27	3.19
78:4	1	1.17	3.61	3.65	3.61	3.67	3.63	3.65

			Linear Models		Markov Models		TAR Models	
Period	NBER	Δy_t	ST	DT	ST	DT	ST	DT
79:1	1	0.03	3.33	3.35	3.36	3.44	3.35	3.33
79:2	1	0.09	3.59	3.51	3.59	3.52	3.63	3.57
79:3	1	0.61	3.63	3.68	3.65	3.68	3.63	3.66
79:4	1	0.19	3.54	3.52	3.54	3.54	3.62	3.59
80:1	1	0.42	3.62	3.65	3.61	3.66	3.67	3.68
80:2	0	-2.60	-1.60	-1.85	-1.51	-1.14	-1.14	-1.45
80:3	1	0.02	3.56	3.61	3.53	3.40	3.35	3.58
80:4	1	1.99	2.01	2.30	1.94	1.91	2.67	2.77
81:1	1	1.36	3.58	3.59	3.59	3.61	3.32	3.61
81:2	1	-0.42	1.99	1.98	1.96	2.33	1.74	2.06
81:3	1	0.52	3.57	3.61	3.60	3.59	3.61	3.63
81:4	0	-1.60	1.67	1.02	1.79	1.61	1.68	1.50
82:1	0	-1.24	2.87	2.52	2.93	2.91	3.33	2.37
82:2	0	0.40	3.50	3.56	3.47	3.34	3.54	3.60
82:3	0	-0.44	3.13	2.87	3.20	3.07	2.36	2.44
82:4	1	0.14	3.49	3.47	3.52	3.56	3.25	3.13
83:1	1	0.63	3.60	3.62	3.60	3.55	3.48	3.51
83:2	1	2.69	1.94	2.32	1.81	1.90	2.84	2.64
83:3	1	1.48	3.61	3.57	3.61	3.62	3.35	3.61
83:4	1	1.70	3.55	3.60	3.54	3.61	3.59	3.59
84:1	1	1.91	3.33	3.46	3.32	3.38	3.40	3.45
84:2	1	1.32	3.60	3.62	3.60	3.68	3.64	3.64
84:3	1	0.54	3.54	3.41	3.52	3.46	3.50	3.45
84:4	1	0.67	3.64	3.62	3.63	3.65	3.66	3.65
85:1	1	0.66	3.64	3.64	3.64	3.66	3.65	3.66
85:2	1	0.78	3.64	3.66	3.65	3.67	3.64	3.66
85:3	1	1.27	3.54	3.60	3.58	3.65	3.48	3.51
85:4	1	0.57	3.57	3.54	3.57	3.58	3.63	3.61
86:1	1	1.31	3.54	3.59	3.55	3.60	3.47	3.50
86:2	1	-0.06	3.19	3.10	3.16	3.18	3.27	3.23
86:3	1	0.57	3.66	3.66	3.65	3.66	3.66	3.67
86:4	1	0.33	3.63	3.61	3.61	3.62	3.68	3.67
87:1	1	0.74	3.66	3.68	3.65	3.69	3.62	3.65
87:2	1	1.23	3.57	3.61	3.59	3.59	3.47	3.51
87:3	1	0.97	3.67	3.68	3.68	3.72	3.69	3.69
87:4	1	1.44	3.56	3.59	3.57	3.57	3.52	3.55
88:1	1	0.64	3.60	3.59	3.61	3.68	3.64	3.62
88:2	1	1.06	3.65	3.68	3.64	3.72	3.65	3.67
88:3	1	0.63	3.67	3.67	3.67	3.70	3.70	3.69
88:4	1	0.95	3.67	3.70	3.67	3.73	3.66	3.68
89:1	1	0.79	3.69	3.70	3.70	3.75	3.71	3.72
89:2	1	0.44	3.62	3.62	3.62	3.66	3.68	3.67
89:3	1	0.00	3.46	3.44	3.42	3.46	3.56	3.54
89:4	1	0.37	3.69	3.70	3.69	3.71	3.71	3.72
90:1	1	0.85	3.66	3.69	3.66	3.71	3.57	3.61
90:2	1	0.38	3.60	3.59	3.59	3.64	3.69	3.67
90:3	0	-0.22	3.21	3.20	3.20	3.27	3.40	3.36
90:4	0	-0.80	2.88	2.83	2.97	2.97	3.21	3.02
91:1	0	-0.52	3.47	3.39	3.50	3.50	3.65	3.44
91:2	1	0.53	3.64	3.69	3.61	3.52	3.63	3.71
91:3	1	0.25	3.60	3.57	3.61	3.59	3.52	3.59
91:4	1	0.02	3.42	3.39	3.46	3.51	3.41	3.44
92:1	1	0.77	3.68	3.71	3.66	3.60	3.69	3.71
92:2	1	0.59	3.70	3.68	3.71	3.71	3.73	3.73
92:3	1	0.87	3.71	3.73	3.70	3.73	3.69	3.72
92:4	1	1.40	3.55	3.60	3.53	3.54	3.44	3.53
93:1	1	0.29	3.43	3.33	3.46	3.40	3.50	3.39
93:2	1	0.59	3.72	3.70	3.72	3.75	3.75	3.74
93:3	1	0.66	3.72	3.73	3.72	3.76	3.72	3.74
93:4	1	1.52	3.35	3.49	3.35	3.47	3.21	3.37
94:1	1	0.82	3.70	3.69	3.70	3.77	3.74	3.72
94:2	1	1.00	3.73	3.74	3.73	3.77	3.74	3.75
94:3	1	0.85	3.74	3.74	3.74	3.79	3.76	3.76